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Homogeneous hypersurfaces in euclidean spaces.

Dedicated to Professor Z. Suetuna on his 60th birthday.

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S. Kobayashi [3] proved that a compact connected homogeneous Riemannian manifold M of dimension n is isometric to the sphere if it is isometrically imbedded in the euclidean space E of dimension $n+1$. In this paper we shall prove that a connected homogeneous Riemannian space M (compact or not) of dimension n is isometric to the Riemannian product of a sphere and a euclidean space if M is isometrically imbedded in the euclidean space E of dimension $n+1$ and the rank of the second fundamental form H is of rank $\neq 2$ at some point.

Manifolds and mappings between them will always be of differentiability class C^{∞}.

1. Preliminaries.

Let M be a connected Riemannian manifold. Assume that there exists an isometric map f of M into a euclidean space E, in which we fix a cartesian coordinate system. f is isometric in the sense that the dual map of the differential f' of f carries the Riemannian metric of E to that of M.

Assigning to a point p of M the A-th coordinate component of $f(p)$, $1 \leqq A \leqq \dim E$, we obtain a function f^A on M. For any vector X tangent to M at x, we denote by Xf the vector tangent to E at $f(x)$ whose A-th component is Xf^A and call Xf the covariant differentiation of f in X. We shall write X for ∇_X or $X^{\mu}\nabla_{\mu}$ in coordinates as long as no ambiguity might be feared. In the same way one can define the covariant differentiation Xf' of the differential f' of f and other objects such as a map of M into the tangent bundle of E or into the isometry group of E. It goes without saying that, when X has x as the origin, Xf' is a linear map of the tangent space M_x to M at x into the tangent space $E_{f(x)}$ for any x in M, and that $Xf=f'X$.

It is easy to see that $(Xf')Y$ is normal to $f(M)$ for any vectors X and Y at a point x. Thus $(Xf')Y$ is a linear combination of the normal vectors

$$(Xf')Y=\sum_{1\leqq t\leqq d} H_t \mathfrak{n}_t ,$$