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Perturbation of continuous spectra by unbounded operators, I.

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§ 1. Introduction and theorems.

1. Introduction. Recently Kato proved in [**5**], among others, that the absolutely continuous part of the spectrum of a self-adjoint operator H_0 is stable under the addition of a bounded self-adjoint perturbation V with finite trace norm. So far as we impose the assumption on V irrespective of H_0, this theorem was shown to be the best possible one in the sense that "trace norm" can not be replaced by any other "cross norm" for bounded operators (Kuroda [**9**]). The main purpose of the present paper is to generalize the above mentioned theorem of Kato in another direction so as to include those unbounded perturbations which are *relatively* small with respect to H_0. In this generalized form we can apply it to some problems of differential operators, especially to the Schrödinger operator of quantum mechanics.

On the other hand, the stability of the continuous spectra is closely connected with the asymptotic properties of the family of unitary operators $\{\exp(itH)\exp(-itH_0)\}$, where H is the perturbed operator, in other words, with the existence of the so-called wave and scattering operators in quantum mechanics. The relations between these two seemingly different concepts are given, for example, in the previous paper of the writer (see Kuroda [**10**] and the references given in [**10**]). According to it, the stability of "continuous spectra" is established if we prove the existence of the wave operators, the definition of which will be given in the next paragraph. We shall study the problem from this point of view. The application of our theorem gives an existence proof of these operators in some problems of quantum mechanics.

2. Unitary equivalence and the wave operator. Let $\mathfrak{H}$ be a Hilbert space and H_0 and H self-adjoint operators in $\mathfrak{H}$; let $\mathfrak{M}_0$ and $\mathfrak{M}$ be the absolutely continuous subspaces of $\mathfrak{H}$ with respect to H_0 and H[1]; and let P_0 and P be

1) For the definition of the absolutely continuous subspace, see e.g. Kato [**4**], Kuroda [**10**]. We agree that a "subspace" always means a closed subspace.