On projective transformations of Riemannian manifolds.

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The main purpose of this paper is to prove the following theorems:

**Theorem 1.** Let $M$ and $M'$ be $n$-dimensional Riemannian manifolds and suppose that $M$ is locally reducible (but $M'$ is not necessarily locally reducible). If there exists a projective transformation of $M$ to $M'$, then

1) the transformation preserves the curvature tensor, or
2) the local homogeneous holonomy group at any point of $M'$ is the proper orthogonal group $O^+(n)$.

**Theorem 2.** Let $M$ and $M'$ be complete Riemannian manifolds. In order that there exist a non-affine projective transformation of $M$ to $M'$, it is necessary that both $M$ and $M'$ be irreducible.

By Theorem 2 and a theorem due to T. Y. Thomas [8] and A. Lichnerowicz [3], we have

**Corollary.** If a complete Riemannian manifold with parallel Ricci tensor admits a non-affine projective transformation, then the manifold is an irreducible Einstein manifold.

Recently, several authors [4], [7] investigated the manifolds with parallel Ricci tensor and T. Nagano [4] proved that a complete Einstein manifold admitting a non-affine projective transformation is the only manifold whose universal covering space is a sphere.

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§ 1. Preliminaries.

Let $M$ and $M'$ be $n$-dimensional Riemannian manifolds and $f$ a diffeomorphism of $M$ to $M'$. For a geometric object $\mathcal{O}$ on $M'$, we denote by $\mathcal{O}'$ the geometric object on $M$ induced from $\mathcal{O}$ by $f$. For instance we denote by

1) In this paper we suppose that the class of differentiability of manifolds and of transformations is not less than 4. If the class is of $C^\infty$, then, in Case 2), the infinitesimal holonomy group of $M'$ is also $O^+(n)$. As to these holonomy groups, see A. Nijenhuis [5].
2) T. Nagano has proved this corollary by a different method.