Journal of the Mathematical Society of Japan Vol. 11, No. 2, April, 1959

On a certain cup product.

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(Received Oct. 4, 1958)

Introduction. Let K be a complex of a form $S^q \cup e^n \cup e^{n+q}$, i.e. a complex obtained from a q-sphere S^q by attaching an n-cell e^n and then an $(n+q)$-cell e^{n+q} where $n-2 \geqq q \geqq 2$. It is clear that the integral cohomology ring of K is as follows:

$$H^0(K) \approx H^q(K) \approx H^n(K) \approx H^{n+q}(K) \approx Z,$$
$$H^i(K) = 0 \qquad i \neq 0, q, n, n+q,$$

where Z denotes the ring of integers.

Let x, y, z denote the cohomology classes carried by e^n, S^q, e^{n+q} respectively. Then there is an integer m determined by $mz = x \cup y$. Let $\alpha \in \pi_{n-1}(S^q)$ denote the homotopy class of a map, $S^{n-1} \rightarrow S^q$, by which e^n is attached to S^q. I. M. James [**5**] described then K as a complex of type (m, α) and proved the following theorem (Theorem (1.8) l.c.).

J. *Let $[\alpha, l_q] \in \pi_{n+q-2}(S^q)$ denote the Whitehead product of α and a generator $l_q \in \pi_q(S^q)$. Then there exists a complex of type (m, α), if and only if $m[\alpha, l_q]$ is contained in the image of the homomorphism $\alpha_* : \pi_{n+q-2}(S^{n-1}) \rightarrow \pi_{n+q-2}(S^q)$ which is induced by α.*

At the end of the introduction of [**5**], James remarks that it is possible to discuss this topic in term of the cohomology invariant of mappings which are defined in [**10**], although his discussion in [**5**] is based on different methods. We shall show in this paper that **J** can be indeed simply and mechanically proved by the cohomology invariant of mappings.

Let L be a complex of a form $S^q \cup e^n$ which is obtained by attaching e^n to S^q. Since the homotopy type of L depends only on the homotopy class of the attaching map, we denote by $L(\alpha)$ the complex L which has a map of the class $\alpha \in \pi_{n-1}(S^q)$ as the attaching map. Then all complexes of type (m, α) have $L(\alpha)$ as a subcomplex.

Now consider a relative functional cup product of a map $g: (E^{n+q-1}, \dot{E}^{n+q-1}) \rightarrow (L(\alpha), S^q)$, where E^{n+q-1} denotes an $(n+q-1)$-cell and $\dot{E}^{n+q-1}$ its boundary. If we denote by $\tilde{x}$ the generator of $H^n(L(\alpha), S^q)$ identified with the cohomology class of $H^n(L(\alpha))$ which is carried by e^n and denote by $\tilde{y}$ the cohomology class of $H^q(L(\alpha))$ which is carried by S^q, then we have $\tilde{x} \cup \tilde{y} = 0$