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A theory of transformation groups on generalized spaces and its applications to Finsler and Cartan spaces.

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One of the main problems in the differential geometry of spaces with given structures is the determination of spaces admitting structure-preserving transformation groups of sufficiently high orders. The problem in generalized spaces, such as non-metric spaces of linear elements, of hyperplane elements or of spreads[1], has been successfully studied, but scarecely the problem in metric spaces, such as Finsler and Cartan spaces[2]. The only one result in Finsler space is due to H. C. Wang [**32**][3], who, by a beautiful group-theoretic method, determined the n-dimensional Finsler spaces admitting a group of motions of order higher than $n(n-1)/2+1$. Now, the author found that this problem could be also treated and solved by the method of tensor calculus for spaces such as Finsler and Cartan spaces, if we could develop the theory of Lie derivatives in the form adapted for the studying of the transformation groups in these spaces, and this could be done from the stand-point of the theory of fibre bundles.

In the present paper we shall give such a development and apply it to determine all the n-dimensional Finsler and Cartan spaces which admit a group of motions of order $n(n-1)/2+1$, for $n\neq 4$.

In Chapter I, we consider a general tensor bundle space to treat Finsler and Cartan spaces simultaneously. For our discussions, we need the theory of linear connections on a tensor bundle space, but such a theory may be obtained by modifying that on spaces of linear elements developed recently by T. Ōtsuki [**26**]. So we refer to [**26**] for the detail. As the modification is very slight, we have noted, as preliminaries, only what will be essentially used in the following. After that, we shall develop the theory of Lie derivatives, as said above, and consider groups of affine transformations on a tensor bundle space.

In Chapter II, we shall state a principle of determining Finsler spaces admitting a transitive group of motions. This principle follows from H. C.

1) [**9**], [**10**], [**11**], [**12**], [**14**], [**15**], [**16**], [**25**], [**36**, Chap. VIII], [**37**], [**38**], [**39**].
2) Cf. [**6**], [**13**], [**15**], [**16**], [**36**, p. 182].
3) His discussions appear also in [**36**, pp. 183–186].