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Harmonic and Killing tensor fields in Riemannian spaces with boundary.

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In a previous paper [5], we have generalized some of the results on harmonic and Killing vector fields stated in Yano and Bochner [6] to the case of Riemannian spaces with boundary.

The purpose of the present paper is to do exactly the same thing for harmonic and Killing tensor fields. The study of harmonic tensor fields in such Riemannian spaces has been already started by Duff and Spencer [3], Conner [2] and Nakae [4].

§ 1. Fundamental formulas.

We consider a compact manifold M which is the closure of an open submanifold of an n-dimensional orientable Riemannian manifold V_n of class C^r $(r\geqq 2)$ with a positive definite metric $ds^2=g_{\mu\lambda}(\xi)d\xi^\mu d\xi^\lambda$ $(\kappa, \lambda, \cdots=1, 2, \cdots, n)$ and which is represented, in a neighborhood of each point on the boundary B of class C^r by $\xi^n\geqq 0$. It follows that B is an $(n-1)$-dimensional compact orientable submanifold (see, Chern [1]). The boundary B is represented locally also by

$$\xi^\kappa=\xi^\kappa(\eta^h) \qquad (h, i, j, \cdots=1, 2, \cdots, n-1) \tag{1}$$

in $U(P)\cap M$, $U(P)$ being a coordinate neighborhood in V_n of a point P on B.

We put $B_i^{\cdot\kappa}=\partial_i\xi^\kappa=\partial\xi^\kappa/\partial\eta^i$, $'g_{ji}=B_j^{\cdot\mu}B_i^{\cdot\lambda}g_{\mu\lambda}$ and denote by N^κ the unit normal to B such that N^κ and $B_1^{\cdot\kappa}, B_2^{\cdot\kappa}, \cdots, B_{n-1}^{\cdot\kappa}$ form the positive sense of M and by $\mathfrak{g}$ and $'\mathfrak{g}$ the determinants formed by $g_{\mu\lambda}$ and $'g_{ji}$ repectively.

Denoting by $'\nabla_i$ the covariant differentiation of van der Waerden-Bortolloti with respect to $'g_{ji}$ along B, equations of Gauss and those of Weingarten can be written respectively in the form

$$'\nabla_j B_i^{\cdot\kappa}=H_{ji}N^\kappa, \tag{2}$$

$$'\nabla_j N^\kappa=-H_j^{\cdot i}B_i^{\cdot\kappa}, \tag{3}$$

where H_{ji} is the second fundamental tensor of B with respect to the normal N^κ.

Now, Stokes' theorem states: We have, for an arbitrary vector field u^κ in M,