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On parallel displacements in Finsler spaces.

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The notion of parallelism was introduced in Finsler spaces by many authors, who gave interesting definitions thereof from various points of view. Most of these parallelisms are derived from some systems of axioms, but the formulas representing them are more complicated than in the case of Riemannian geometry. So it seems to be interesting that we return to the foundation of the theory and consider whether the complications of the formulas are essentially unavoidable or not.

In this note we shall investigate what conditions are to be imposed on connections in Finsler spaces in order that the analogous properties still hold in our spaces as in Riemannian geometry. As the results of this consideration we shall get a characterization of a Finsler metric among more general metrics (Theorem 3) and it will be shown that the connection defined by E. Cartan [4] as stated in Theorem 1 is the shortest and the fittest from our standpoint (Theorem 2 and 13°). Further, we can immediately derive from it the parallelisms defined by other authors. The parallel displacements defined by J. L. Synge [1], J. H. Taylor [2] and W. Barthel [7] are special cases of the one by E. Cartan, and the connections defined by H. Rund [5], [6] and L. Berwald [3] are immediately obtained from it by applying Lemma 2 and Lemma 3 respectively; and these lemmas give us two methods to define new connections from a given one (Theorem 4).

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§ 1. Spaces with line-elements.

1°. When we deal with the geometry of the differentiable manifold $\mathfrak{F}$ such that its properties depend not only on the points of $\mathfrak{F}$ but also on the non-zero tangent vectors at each point, we shall call $\mathfrak{F}$ the *space with line-elements* and the tangent vector the *supporting element*.

The set of all non-zero tangent vectors at the point p is called the *tangent space at* p and denoted by T_p. The set $\mathfrak{B}(\mathfrak{F})$ of all non-zero tangent vectors on $\mathfrak{F}$ is considered as a differentiable manifold, which is called the *tangent bundle.* As is well known, the tangent vector X at the point p is locally