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A unique continuation theorem for solutions of a parabolic differential equation.

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Introduction.

It was shown by N. Aronszajn [1], [2] that, if $u(x)$ satisfies a second order linear elliptic differential equation $Au(x)=0$ on a domain D and has a zero point of infinite order in D, then it vanishes identically in D. Recently one of the authors has proved a similar result for a parabolic equation $\partial u(t,x)/\partial t=Au(t,x)$ $(0<t<\infty, x\in D)$ for the case when D is bounded. The purpose of this paper is to extend this result to the case when D is not necessarily bounded.

§ 1. Assumptions and the main theorems.

Let D be a (not-necessarily bounded) domain in a euclidean m-space whose boundary $B=\bar{D}-D$ consists of at most countably many C^3-hypersurfaces of $m-1$ dimension. Consider an elliptic differential operator A defined by

$$\text{(A)} \qquad Au=\frac{1}{\sqrt{a(x)}}\frac{\partial}{\partial x^i}\left(\sqrt{a(x)}\,a^{ij}\frac{\partial}{\partial x^j}u\right)+c(x)u \qquad \text{for } x\in D$$

with a boundary condition

$$\text{(B)} \qquad \alpha(\xi)u+\{1-\alpha(\xi)\}\partial u/\partial n_\xi=0 \qquad \text{for } \xi\in B.$$

Here $\|a^{ij}(x)\|$ denotes a strictly positive-definite symmetric matrix for any $x\in\bar{D}$, $0\leqq\alpha(\xi)\leqq 1$ on B, $\partial^2 a^{ij}(x)/\partial x^k\partial x^l$ $(i,j,k,l=1,\cdots,m)$ and $\partial^2\alpha(\xi)/\partial\xi^p\partial\xi^q$ $(p,q=1,\cdots,m-1)$ are Lipschitz continuous in $x\in\bar{D}$ and in $\xi\in B$ respectively, where local coordinates on B are denoted by $<\xi^1,\cdots,\xi^{m-1}>$.

Moreover $c(x)$ is assumed to be Lipschitz continuous in $x\in\bar{D}$, and satisfies

$$\text{(C)} \qquad -\infty<c(x)\leqq C<\infty$$

for some constant C. Here the differentiability of functions on $\bar{D}$ at any point $\xi\in B$ and normal derivatives $\partial u/\partial n_\xi$ $(\xi\in B)$ with respect to the metric tensor $a^{ij}(x)$ should be understood as those defined in one of Itô's papers [6].

Under these assumption shown above, it was shown in [6] that there exists a so-called fundamental solution $U(t,y,x)=U(t,x,y)\geqq 0$ of a parabolic equation