Journal of the Mathematical Society of Japan Vol. 10, No. 2, April, 1958

On linear Lie algebras I.

By Yosio MUTŌ

(Received Nov. 5, 1957)

Introduction

Let $\mathfrak{gl}(R^n)$ be the set of all linear transformations X of an n-dimensional linear space R^n over the field of real numbers, equipped with the ordinary law of addition $X+X'$ and the law of composition $[X, Y]=XY-YX$. The purpose of the present paper, which is considered as the first of the series of papers pursuing the same purpose, is to deal with a method of studying some properties of r-dimensional subalgebras $\mathfrak{g}$ of this general linear Lie algebra $\mathfrak{gl}(R^n)$.

If we take a base S composed of a set of n linearly independent vectors $\boldsymbol{e}_\lambda$ $(\lambda=1,\cdots,n)$ of R^n, then a subalgebra $\mathfrak{g}$ is represented by an r-dimensional linear subspace in an n^2-dimensional linear space spanned by all matrices of degree n. This subspace will be denoted by $\mathfrak{K}(\mathfrak{g}, S)$, or by $\mathfrak{K}$ if there is no possibility of confusion. If we take another base $\tilde{S}(\tilde{\boldsymbol{e}}_\lambda)$ such that

$$\hat{S}=AS, \quad \tilde{\boldsymbol{e}}_\lambda=A^{\alpha}_{\cdot\lambda}\boldsymbol{e}_\alpha,$$

then a matrix K of $\mathfrak{K}$ representing an element of $\mathfrak{g}$ is transformed into $\tilde{K}=A^{-1}KA$. This fact will be denoted by

$$\mathfrak{K}(\mathfrak{g}, AS)=A^{-1}\mathfrak{K}(\mathfrak{g}, S)A.$$

The set of matrices $(V^{\lambda}_{\cdot\mu})$ where $V^{\lambda}_{\cdot\mu}$ satisfy

$$K^{\lambda}_{\cdot\mu}V^{\mu}_{\cdot\lambda}=0$$

for all matrices $(K^{\lambda}_{\cdot\mu})$ of $\mathfrak{K}$ spans an (n^2-r)-dimensional linear subspace in an n^2-dimensional linear space spanned by all matrices of degree n. This subspace will be denoted by $\mathfrak{B}(\mathfrak{g}, S)$ or by $\mathfrak{B}$ for short. The law of transformation is

$$\mathfrak{B}(\mathfrak{g}, AS)=A^{-1}\mathfrak{B}(\mathfrak{g}, S)A.$$

$\mathfrak{B}$ may be considered to represent a set of transformations of R^n which will be denoted by $\mathfrak{t}(\mathfrak{g})$ or by $\mathfrak{t}$ for short.

If we take a suitable base S in R^n and moreover, if we take a suitable base M in $\mathfrak{B}(\mathfrak{g}, S)$

$$M: \underset{A}{V} \qquad (A=1,\cdots,m;\ m=n^2-r),$$