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# On the local property of the absolute summability $|C, \alpha|$ for Fourier series.

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**1.** V. A. Magarik [4] has generalized Wiener's theorem[2)] on the absolute convergence of Fourier series to the absolute summability $|C, \alpha|$. His assertion is as follows:

*Let $f(x)$ be Lebesgue integrable in the interval $(-\pi, \pi)$ and periodic with period $2\pi$. If at every point $y$ on the closed interval $[-\pi, \pi]$ there are a function $g_y(x)$ and a $\delta>0$ such that* (i) *$g_y(x)=f(x)$ for $|x-y|<\delta$, and* (ii) *both the Fourier series of $g_y(x)$ and its conjugate series are absolutely summable $|C, \alpha|$,*[3)] *then the Fourier series of $f(x)$ is absolutely summable $|C, \alpha|$, where $\alpha \geqq 0$.*

For the case $\alpha=1$, W. C. Randels [5] proved this proposition without the condition on the absolute summability $|C, 1|$ for the conjugate series.

In the present note, we shall show that the condition on the absolute summability for the conjugate series is also superfluous for the general case; that is, the following theorem will be established.

THEOREM. *Let $f(x)$ be Lebesgue integrable in the interval $(-\pi, \pi)$ and periodic with period $2\pi$. If at every point $y$ on the closed interval $[-\pi, \pi]$ there are a function $g_y(x)$ and a $\delta>0$ such that* (i) *$g_y(x)=f(x)$ for $|x-y|<\delta$ and* (ii) *the Fourier series of $g_y(x)$ is absolutely summable $|C, \alpha|$, then the Fourier series of $f(x)$ is absolutely summable $|C, \alpha|$, where $\alpha \geqq 0$.*

**2.** The case for $\alpha>1$ of our theorem follows immediately from the known theorem of L. S. Bosanquet [1]:

*The absolute summability $|C, \alpha|$, $\alpha>1$, for Fourier series of a Lebesgue integrable function with period $2\pi$ at a point $x=x_0$ depends only on the behaviour of the generating function in the neighbourhood of the point $x_0$.*

On the other hand, L. S. Bosanquet and H. Kestelman [2] proved that *the mentioned result of L. S. Bosanquet does not hold for $\alpha=1$.*

Thus, it is the case $0 \leqq \alpha \leqq 1$ in which we are interested. However, it

1) The author wishes to thank Dr. S. Yano for his valuable advice during the preparation of this paper.

2) A. Zygmund [6], p. 140.

3) For the definition of absolute summability $|C, \alpha|$, see below.