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# Homeomorphy classification of total spaces of sphere bundles over spheres.

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## Introduction.

The homeomorphy problem, i. e. the problem to determine whether two given topological spaces are homeomorphic or not, has been approached from various directions. Although we are as yet far from the general solution of this problem, even for the case where the given spaces are differentiable manifolds, the recent developments of the homotopy theory and of characteristic classes seem to form significant contributions to the homeomorphy theory of differentiable manifolds.

The present paper attempts a step in the homeomorphy theory of differentiable manifolds. We shall consider here namely total spaces of 3-sphere bundles over the 4-sphere and those of 7-sphere bundles over the 8-sphere, and shall explicitly give homeomorphic maps between some of these spaces. We shall see that these spaces have the same Pontrjagin classes (with respect to differentiable structures defined naturally), and also that, conversely, the spaces under our consideration (under some restrictions, see Theorems 3.4 and 3.5) which have the same homotopy type and the same Pontrjagin classes are homeomorphic. These results would offer some interesting facts to the problem of "topological invariance of Pontrjagin classes".

As an application we shall obtain the *homotopy* classification of the sphere bundles over spheres which have no cross section. And also we are able to generalize Milnor's result (Milnor [3]), that is, we shall obtain further examples of 7-dimensional and 15-dimensional manifolds which are homeomorphic but not diffeomorphic.

Notations and terminologies used in the paper are made clear in section 1. Section 2 contains a remark on Pontrjagin classes of sphere bundles over spheres computed in the previous paper (Tamura [5]), and Pontrjagin classes of the Cayley projective plane are obtained as a corollary.

In section 3, main part of this paper, we construct maps which induce homeomorphisms between total spaces of 3-sphere bundles over the 4-sphere and between total spaces of 7-sphere bundles over the 8-sphere. The total