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On the partitions of a number into the powers of prime numbers.

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1. In **1953**, Szekeres [4] proved an asymptotic formula for the number $P(n,m)$ of the partitions of n into positive integers not exceeding m, for large n and m. The generating function of $P(n,m)$ is

$$F(w)=\prod_{\nu=1}^{m}(1-w^{\nu})^{-1}=\sum_{n=0}^{\infty}P(n,m)w^{n} \qquad (|w|<1)$$

and we have

$$P(n,m)=\frac{1}{2\pi}e^{n\rho}\int_{-\pi}^{\pi}F(e^{-\rho+i\theta})e^{-ni\theta}d\theta\,, \tag{1}$$

where ρ is the root of the equation

$$n=\sum_{\nu=1}^{m}\frac{\nu}{e^{\nu\rho}-1}\,.$$

The essential point in Szekeres's proof is this determination of ρ, by which it is shown that the integral over the neighborhood of the point $\theta=0$ in (1) gives the principal term of the asymptotic formula for $P(n,m)$.

In this paper, we shall prove, by a method partly analogous to Szekeres's proof, an asymptotic formula for the number $T(n,m;k)$ of the partitions of n into k-th $(k\geqq 1)$ powers of *prime numbers* not exceeding m. Our result is stated as follows:

THEOREM. *Let n and m be sufficiently large integers and $n^{1/k}\geqq m$. Then we have, uniformly in n and m,*

$$T(n,m;k)=\frac{1}{\sqrt{2\pi A_2}}e^{n\alpha+A_1}\left\{1+O\left(\max\left(n^{-\frac{k}{2(k+1)(k+2)}},\,m^{-\frac{k}{2(k+2)}}\right)\right)\right\},$$

where α is the root of equation

$$n=\sum_{p\leqq m}\frac{p^{k}}{e^{\alpha p^{k}}-1} \tag{2}$$