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Cohomology mod p of the p-fold symmetric products of spheres.

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1. Introduction

In this note we shall study the cohomology modulo p of the p-fold symmetric product $\mathfrak{S}_p(S^n)$ of an n-sphere S^n $(n \geqq 1)$, where p is an odd prime. Let Z_p denote the group of integers modulo p. As usual, denote by $\mathcal{P}^s$ the reduced p-th power, by Δ_p the Bockstein homomorphism and by $\smile$ the cup product. Then our main result is stated as follows:

THEOREM 1. *We have* $H^n(\mathfrak{S}_p(S^n); Z_p) \approx Z_p$. *Denote by h a generator of* $H^n(\mathfrak{S}_p(S^n); Z_p)$. *Then a set of generators for the cohomology group* $H^*(\mathfrak{S}_p(S^n); Z_p)$ *can be formed with all elements of the following four types:*

(I) $\mathbf{1}$,[1] (II) $\mathcal{P}^s(h)$ $(0 \leqq s \leqq n/2)$, (III) $\Delta_p \mathcal{P}^s(h)$ $(1 \leqq s < n/2)$,

(IV) h^q $(2 \leqq q < p)$, *where* $h^q = h \smile h \smile \cdots \smile h$ *(q factors).*

Define $B(n, p)$ *as a set of all elements of the above types* (I)$\sim$(IV) *or* (I)$\sim$(III) *according as n is even or odd. Then* $B(n, p)$ *is linearly independent.*

We shall also calculate the cup products and the reduced powers in $H^*(\mathfrak{S}_p(S^n); Z_p)$ (§ 5).

Our proof depends on the results about the cohomology of the p-fold cyclic product $\mathfrak{Z}_p(S^n)$ of S^n, which I have obtained in the paper [4],[2] together with the technique which was used by Steenrod to prove Theorem 4.8 in his paper [7].

Throughout this paper, the coefficient group is always Z_p, and hence we shall hereafter omit to mention it.

2. Symmetric, cyclic, cartesian products

Denote by $\mathfrak{S}_p$ the symmetric group of the letters $1, 2, \cdots, p$. Let $t \in \mathfrak{S}_p$ be an element defined by $t(j) = j+1 \bmod p$ $(j = 1, 2, \cdots, p)$, and

1) $\mathbf{1}$ denotes the unit cohomology class.

2) Numbers in square brackets refer to the bibliography at the end of the paper.