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Class formations IV.

(Infinite extension of the ground field)

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Let K_0^* be an infinite algebraic extension of the rational number field (or of the p-adic number field). We may expect that a kind of analogy of class field theory would hold for finite abelian extensions over K_0^*. In 1936–37 M. Moriya has established in his papers [7], [8] such a theory over K_0^* for those finite abelian extensions K^*/K_0^* for which the degrees $[K^*:K_0^*]$ are relatively prime to each one of a certain set of prime numbers determined by K_0^*. Recently M. Mori [9] considered the same problem (in local case) from a different point of view and obtained similar results as in the theory of Moriya without any restriction on the degrees of abelian extensions K^*/K_0^*.

The purpose of the present paper is to consider an analogous problem in the framework of class formation theory. Let $\{A(K); K\in\mathfrak{K}\}$ be a given class formation over a ground field k_0 where $\mathfrak{K}$ is the set of all finite extensions of k_0 contained in a fixed infinite normal algebraic extension Ω/k_0 and $A(K)$ is the abelian group attached to K $(K\in\mathfrak{K})$. Furthermore, we assume that every $A(K)$ is a compact topological group. Now let K_0^* be an arbitrary infinite extension of k_0 contained in Ω. Then let us take $\mathfrak{K}^*=\{K^*; K_0^*\subset K^*\subset\Omega, [K^*:K_0^*]<\infty\}$ and let $A^*(K^*)$ be the inverse limit group of $\{A(k_\lambda); k_\lambda\subset K\}$. Our main result is that $\{A^*(K^*); K^*\in\mathfrak{K}^*\}$ is a class formation over the ground field K_0^* (Theorem 1). We can apply this result also to the cases of local and global class field theory by taking a suitable class formation in each case (Theorem 5, 7). In particular, our results coincide with that of M. Mori in local case. In a previous paper [4] the author considered the same problem in a class formation after the method of M. Moriya. In §6 we shall consider the relation between our present method and that used before in [4].