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On similarities and isomorphisms of ideals in a ring

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As for the connections between similarities and isomorphisms of ideals, the investigations known up to this time seem to be restricted only to the case of principal ideal domains. In the present paper we shall study some relations between these two notions of equivalence for ideals for the case of a ring which has a unit element and satisfies the minimum (whence the maximum) condition for left and right ideals. This problem was suggested to me by Prof. K. Morita and I express my hearty thanks to him.

In Section 1, we shall establish our main theorem which asserts that a left [right] similarity between two left [right] ideals implies a left [right] operator-isomorphism of them. In Section 2, we shall deal with the problem: in what ring does every left [right] operator-isomorphism between two left [right] ideals imply a left [right] similarity of them? For the validity of this implication, we shall show that it is not necessary but sufficient that the ring be quasi-Frobeniusean.

Throughout this paper "isomorphism" will mean "operator-isomorphism".

1. Let A be a ring with a unit element 1 satisfying the minimum (whence the maximum) condition for left and right ideals, and let N be its radical. Then we have the following.[1)]

THEOREM 1. *Let L and L' be left ideals of A. If L is left similar to L', that is, the residue class module A/L is A-isomorphic to the residue class module A/L', then L is A-isomorphic to L'. Furthermore, if, by this similarity, the residue class* 1 *(mod L) is mapped onto the residue class a (mod L'), then there exists a regular element a_0 of A such that $a_0 \equiv a$ (mod L') and $L' = La_0$. Convsersely if there exists a regular elements a_0 of A such that $L' = La_0$, then L is left similar to L'. The same is of course true for right ideals.*

1) Recently the most general extension of our Theorem 1 has been obtained in [5].