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# $L$-functions of number fields and zeta functions of abelian varieties.

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## Introduction.

It was found out in several cases that Hasse's zeta functions of algebraic curves or of abelian varieties over an algebraic number field can be expressed by Hecke's $L$-functions with "Grössencharaktere" of that field.[1] It deserves our attention that these phenomena have always presented themselves in connection with arithmetic of abelian extensions of that number field. However, since the relation of Hasse's functions with abelian extensions was not so direct in the proofs of these results, which have been done from different angles, it would be desirable to clearify the relation between Hasse's functions and abelian extensions attached to abelian varieties in question, treating all cases from a unified point of view. This is the first problem. In pursuing this problem, I have succeeded in obtaining a new interpretation of Hasse's functions in general, and in characterizing under which particular conditions the above phenomena take place.

On the other hand, "Grössencharaktere" can be interpreted as characters of idèle class groups, so it seems natural that they have some connection with abelian varieties related to abelian extensions of the basic fields. However, as class field theory shows, it is not the idèle class group, but the factor group of it by the connected component of the identity, that can be interpreted by the Galois group of the maximally abelian extension. The above phenomena suggest conversely the possibility of an interpretation of characters of idèle class group by something connected with abelian extensions. To find out such an interpretation is a problem, first proposed by A. Weil [4], which seems no less important than the above.

1) Weil [9], Deuring [1], Taniyama [3]. There are also cases first treated by Eichler, where this does not hold.