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On the representation of complemented modular lattices.

By Ichiro AMEMIYA

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J. von Neumann [1] has established a beautiful theory of representation of complemented modular lattices, resulting in a generalization of the coordinatization theorem of the projective geometry to the case of the complemented modular lattice with homogeneous basis of degree $\geqq 4$. His theory is presented in a book [2] of F. Maeda, where simplification of proofs obtained by Kodaira and Huruya [3] is taken into accout. However, there still remain considerable difficulties in the construction of the auxiliary ring, and also in the final step of the induction to attain the regular ring representation of the lattice. The purpose of this paper is to simplify further this theory so as to obtain the same results through proofs which present no such difficulties.

Our method is based on the fundamental theorem in § 1 which asserts the existence of the lattice-automorphisms of a certain type. In § 2, we shall construct certain automorphism groups of the lattice and investigate the relations among these groups which will lead us, in § 3, naturally to the definition of the auxiliary ring. In § 4, we shall attain the coordinatization theorem; we shall meet with no 'final step difficulty' (cf. footnote (5)).

To write this paper the author has had frequent consultations with the book of F. Maeda [2]. He also wishes to express his hearty thanks to Professor S. Iyanaga for his encouragement and advices.

§ 1. Fundamental theorem.

Let L be a complemented modular lattice throughout this paper.

First we shall introduce some notions analogous to those used in the combinatorial topology. Let s and c be two elements of L such that $s \neq 1$[1] and $s \geqq c$. These elements s, c will be fixed once for all

1) 1 denotes the maximum element of L, and 0 the minimum element.