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A note on one-parameter and monothetic groups.*

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An important technique in the study of locally compact groups is the construction of one-parameter subgroups within the group. Such subgroups are topological analogues of the cyclic subgroups, which play such an important role in abstract group theory. The principal fact about one parameter subgroups of locally compact groups is that they are either isomorphic and homeomorphic to the reals, or else they have compact closure. The corresponding fact about cyclic subgroups is almost identical: a cyclic subgroup of a locally compact group is either discrete or has compact closure. (This latter fact includes the usual facts about cyclic subgroups of abstract groups, since any group is a topological group if given the discrete topology.)

Several proofs of these results have been given in the literature (in particular, see [3] and [5]). The purpose of this note is to prove a slight generalization of these facts. The proof to be given is not, however, simply a generalization of the extant proofs. Rather, it is based on certain concepts introduced in an earlier paper [7], and thus becomes, in a sense, more intrinsic and perhaps more conceptual.

A remark is in order concerning the terminology employed in this note. The three definitions stated in the sequel will give meaning to certain expressions used throughout the paper. The reader is cautioned that this usage is not universal. In general, the common practice is to give a more restricted sense to some of these terms.

DEFINITION 1. Let R denote the additive group of real numbers, and let Y be any subgroup of R. A topological group H is called a one-parameter group if there exists a continuous homomorphism f of Y onto H. In this case, the group Y is called a parameter for H, and the mapping f is called a parametrization of H by Y. If a subgroup H of a topological group G is a one-parameter group, then

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