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Remark on my paper: On Skolem's theorem.

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It was proved in my paper [1] that if the system of axioms of Fraenkel-von Neumann of the set-theory is consistent, then system remains consistent after addition of the axiom that every set is a univalent image of ω. This result was established by Theorems 1, 2 of that paper. I should like to remark now that from the proof of these theorems follows immediately also the following result:

"Let Γ_1 be any consistent system of axioms in Gentzen's *LK*, representing mathematically a certain domain D of elements. Then Γ_1 remains consistent after addition of the system of axioms of the theory of natural numbers, and the axiom that every element of D is a univalent image of a natural number.

I shall formulate this result more precisely in the following lines, and indicate how to prove it.

We begin with Γ_a, the system of axioms of "arithmetic" consisting of axioms of the theory of natural numbers except the axiom of mathematical induction. In this paper, Γ_a means the following axioms:

$\forall x(x=x)$

$\forall A\forall x\forall y(x=y \rightarrow (A(x)\leftrightarrow A(y)))$ (See [3], § 1 for the notation $\forall A$.)

$\forall x\forall y\forall z(x<y \wedge y<z \rightarrow x<z)$

$\forall x\forall y \neg (x=y \wedge x<y)$

$\forall x\forall y(x<y \rightarrow x'<y \vee x'=y)$

$\forall x(x<x')$

$\forall x(0<x \vee 0=x)$

$\forall x(x+0=x)$

$\forall x\forall y(x+y'=(x+y)')$

$\forall x\forall y(x+y=y+x)$

$\forall x\forall y\forall z((x+y)+z=x+(y+z))$

$\forall x\forall y(x<y \leftrightarrow \exists z(0<z \wedge x+z=y))$

$\forall x\forall y(x\cdot y=y\cdot x)$

$\forall x\forall y\forall z((x+y)\cdot z=x\cdot z+y\cdot z)$