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On the number of prime factors of integers II.

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1. Introduction.

Let P be the set of all rational prime numbers, and $\{\pi_1,\cdots,\pi_k\}$ a family of subsets of P satisfying the following conditions:

(C_1) The sets $\pi_1,\cdots,\pi_k$ are mutually disjoint;

(C_2) The series $\sum_{p\in\pi_i}\frac{1}{p}$ $(i=1,\cdots,k)$ are divergent.

We need not suppose $\pi_1\cup\cdots\cup\pi_k=P$ for the following development. We shall suppose, except for in the last section, the family $\{\pi_1,\cdots,\pi_k\}$ as given once for all. The letter i will always represent one of the integers $1,\cdots,k$.

We denote by $\omega_i(n)$ the number of distinct prime factors of a positive integer n which belong to the set π_i:

$$\omega_i(n)=\sum_{p\mid n,\, p\in\pi_i}1\,.$$

We also put

$$y_i(n)=\sum_{p\leqq n,\, p\in\pi_i}\frac{1}{p}\,,$$

and denote by n_0 the least positive integer for which $y_i(n_0)>0$ $(i=1,\cdots,k)$.[1] We further put, for $n\geqq n_0$,

$$u_i(n)=\frac{\omega_i(n)-y_i(n)}{\sqrt{y_i(n)}}\,.$$

Then, to each integer $n\geqq n_0$, there corresponds a point $U(n)=(u_1(n),\cdots,u_k(n))$ in the space R^k of k dimensions. Let E be a Jordan-measurable set, bounded or unbounded, in R^k, and let $A(x;E)$ denote the number of integers $n, n_0\leqq n\leqq x$, for which the corresponding points $U(n)$ belong to the set E.

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1) When it is desirable to emphasize that we are considering the relevant formulas for $i=1,\cdots,k$ simultaneously, we add the expression '$(i=1,\cdots,k)$' to indicate the simultaneousness.