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Topological structures in ordered linear spaces.

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Introduction. An adequate tool for the treatment of the integration theory is, as is well-known, the theory of ordered linear spaces (cf. [3]). These spaces have several topological structures, each of which has its own significance, and it is of interest from the view point of general analysis to investigate the relation among them. Such investigations have been made by H. Nakano [11] and [13], and by G. Köthe [9] in some special cases, among others. The purpose of the present paper is to generalize the previously obtained results in this regard and present them in a simplified form.

This paper is divided into 8 articles. In § 1, we shall give a certain property of complete lattices, which is fundamental for our paper. In § 2, the *intrinsic* topology of lattices is defined and determined for some particular cases, Boolean lattices etc. § 3 is devoted to the proof of the completeness of some uniform structures of lattices. We confine ourselves to ordered linear spaces in the next three articles, where the weakest and strongest compatible topologies are given and the compatibility of Mackey's topology in Nakano's duality is proved. In § 7, we given a topological characterization of some atomic lattices, for instance, the spaces considered in G. Köthe [9]. The last article presents some detailed considerations and pathological examples.

Preliminaries. Let L be a lattice. A subset $\{a_\lambda; \lambda\in\Lambda\}$ of L is said to be *upper* (resp. *lower*) *directed*, if for any $\lambda, \lambda'\in\Lambda$, there exists $\lambda''\in\Lambda$ such that $a_{\lambda''}\geqq a_\lambda, a_{\lambda'}$ (resp. $a_{\lambda''}\leqq a_\lambda, a_{\lambda'}$) and we indicate it by $a_\lambda\uparrow_{\lambda\in\Lambda}$ (resp. $a_\lambda\downarrow_{\lambda\in\Lambda}$). If, moreover, $\bigcup_{\lambda\in\Lambda} a_\lambda=a$ (resp. $\bigcap_{\lambda\in\Lambda} a_\lambda=a$) we write $a_\lambda\uparrow_{\lambda\in\Lambda} a$ (resp. $a_\lambda\downarrow_{\lambda\in\Lambda} a$). The interval $\{x; a\leqq x\leqq b\}$ of L is denoted by $[a, b]$. $A\subset L$ is said to be *order-bounded*, if $A\subset[a, b]$ for some a and b. A is said to be *order-convex*, if $a, b\in A$ imply $[a, b]\subset A$. When L is complete, the greatest (resp. least) element is denoted by 1 (resp. 0). When L is a Boolean lattice, the complement of x and symmetric difference between x and y are denoted resp. by $1-x$ and $x\Delta y$.