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Some theorems in dimension theory for non-separable spaces.

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This paper provides some theorems in dimension theory for non-separable spaces. Let R be a topological space, $\dim R$ the covering dimension of R, $\operatorname{ind} R$ the so-called "small" inductive dimension of R defined by means of boundaries of neighborhoods of points, and $\operatorname{Ind} R$ the so-called "large" inductive dimension of R defined by means of boundaries of neighborhoods of closed sets. (Cf. [14]. In the notations of [2], Appendix, p. 153, we have $\operatorname{ind} R=d_1(R)$, $\operatorname{Ind} R=d_2(R)$.) It is to be noted that when R is normal, $\dim R\leqq n$ is equivalent to the following condition: For any closed subset C of R and for any mapping ($=$continuous transformation) f from C into an n-sphere there exists a continuous extension g of f defined on the whole space R.

In § 1 we shall give the sum theorem of covering dimension for metric spaces which is a generalization of the known. In § 2 we shall study closed mappings which lower dimension and related problems. In § 3 we shall give a new definition of dimension-kernel and shall study some properties concerning it.

§ 1. Sum theorem of covering dimension.

Let R be a topological space and $\mathfrak{U}=\{U_\alpha;\alpha\in A\}$ be a collection of subsets of R. Then $\mathfrak{U}\cap S$, S being a subset of R, stands for $\{U_\alpha\cap S;\alpha\in A\}$.

LEMMA 1. *Let S be a closed subset of a normal space R and $\mathfrak{U}=\{U_\alpha;\alpha\in A\}$ be a finite open covering of S whose elements are* F_σ. *Then there exists a finite open collection $\mathfrak{V}=\{V_\alpha;\alpha\in A\}$ of R whose elements are* F_σ *such that the order of $\mathfrak{V}$ is not greater than that of $\mathfrak{U}$ and $V_\alpha\cap S=U_\alpha$ for every $\alpha\in A$.*

LEMMA 2. *An* F_σ*-subset of a normal space is also normal as a relative space.*