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# Minimal complexs of fibre spaces.

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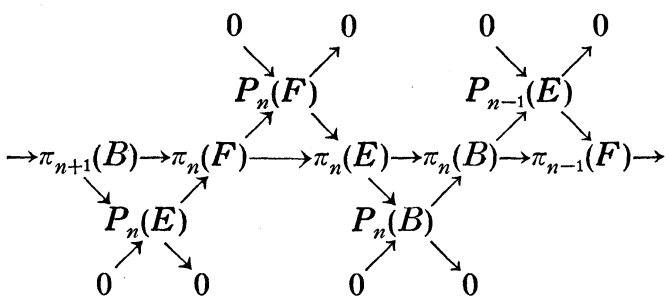
In this note we shall give the algebraic description of several fibre spaces introduced by H. Cartan and J. P. Serre. Our general idea may be characterized as follows. Let $(E, p, B, F)$ be a fibre space in the sense of Serre, and let its homotopy exact sequence be as follows




If we denote the minimal complex of $E, B$ and $F$ with $K(E)$, $K(B)$ and $K(F)$. Then we have an expression of the form

$$\begin{array}{llllll} & K(F) & \cdots\times K(P_n(E), n) & \overset{\bar{e}^{n+1}}{\times} K(P_n(F), n) & \overset{\bar{k}^{n+1}}{\times} K(P_{n-1}(E), n-1) & \times\cdots \\ (*)K(E)\simeq & \times c \equiv & \times u^{n+1} & f^{n+1} \quad \times e^{n+1} & c^{n+1} \quad \times u^{n} & \times\cdots \\ & K(B) & \cdots\times K(P_n(E), n+1) & \underset{\underline{k}^{n+2}}{\times} K(P_n(B), n) & \underset{\underline{e}^{n+1}}{\times} K(P_{n-1}(E), n) & \times\cdots \end{array}$$

where $K(\pi, n)$ denotes the Eilenberg-McLane complex and the meanings of notations $u^{n+1}, c^{n+1}, k^{n+1}$, etc. are to be explained later (cf. § 3). $K(F)\overset{c}{\times}K(B)$ may be considered as a fibre bundle with the fibre

$$K(B)\equiv \quad \cdots\times K(P_n(E), n+1) \overset{\underline{k}^{n+2}}{\times} K(P_n(B), n) \overset{\underline{e}^{n+1}}{\times} K(P_{n-1}(E), n) \quad \times\cdots$$

over the base

$$K(F)\equiv \quad \cdots\times K(P_n(E), n) \quad \overset{\bar{e}^{n+1}}{\times} K(P_n(F), n) \overset{\bar{k}^{n+1}}{\times} K(P_{n-1}(E), n-1)\times\cdots$$