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On certain cohomological operations.

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Introduction

Let A, B be given abelian groups and m, n fixed non-negative integers. Then Serre [7] has defined as follows the cohomology operation relative to (A, B, m, n). It is a mapping C defined for each CW-complex K of the m-th cohomology group $H^m(K, A)$ into $H^n(K, B)$, such that the following diagram is commutative

$$\begin{array}{ccc} H^m(K, A) & \xrightarrow{f^*} & H^m(K', A) \\ \downarrow C & & \downarrow C \\ H^n(K, B) & \xrightarrow{f^*} & H^n(K', B) , \end{array}$$

where K' is another CW-complex, f^* the homomorphism of the cohomology group of K into that of K' induced by a simplicial mapping $f: K' \rightarrow K$. In generalizing this notion, we shall now consider operations of the following kind. Our mapping C has as its domain of definition a subgroup S of $H^m(K, A)$ and as its range a factor group $H^n(K, B)/M$ of $H^n(K, B)$. Once C is given, an subgroup $S = S(K)$ of $H^m(K, A)$ and the subgroup $M = M(K)$ of $H^n(K, B)$ are thus defined by K; we postulate now

$$S(K') \subset f^*(S(K)) ,$$
$$M(K') \subset f^*(M(K))$$

for every simplicial mapping $f: K' \rightarrow K$. C will be then called cohomological operation if the following diagram is commutative

$$\begin{array}{ccc} H^m(K, A) \supset S & \xrightarrow{f^*} & S' \subset H^m(K, A') \\ \downarrow C & & \downarrow C \\ H^n(K, B)/M & \xrightarrow{f^*} & H^n(K', B)/M' , \end{array}$$