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On universal tensorial forms on a principal fibre bundle.

By Yoshihiro TASHIRO

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The concept of the connection of generalized spaces due to E. Cartan has been recently clarified by several authors in the light of the notion of fibre bundles. In particular, S. S. Chern [3], [4] and Ambrose-Singer [1] have generalized the covariant differentiation of tensors and tensorial forms in affinely connected manifolds to the case of principal fibre bundles with connection. S. S. Chern [3] has shown thereby in the case of affine connection that tensorial forms on the base space are in one-one correspondence with certain forms on the bundle of frames with some characteristic properties. In this paper, we shall generalize this result to the case of any principal fibre bundle with connection. After preliminaries (§ 1), we shall define namely (in § 2) the *universal tensorial forms* on the bundle space which are in one-one correspondence with tensorial forms on the base space (Theorem 1). The covariant differential in Ambrose-Singer's sense of a universal tensorial form will be given by an explicit formula (Theorems 2, 3). Finally we shall give a useful characterization (Theorem 4) of the universal tensorial form by means of the covariant differentiation, generalizing the results of S. S. Chern [4] and Boothby [2].

§ 1. Preliminaries on connection. Let $\mathcal{B}=\{B, X, G, G\}$ be a differentiable principal bundle. Thus we assume that the bundle space B and the base space X of B are differentiable spaces, the fibre G (indicated by the first G) and the structural group G (indicated by the second G) are the same Lie group, and that the projection p, and the coordinate functions $\varphi_\alpha \in \Phi$, are differentiable maps.[1)]

1) Throughout this paper we shall always assume that spaces and maps are of differentiability of a suitable high class.