Note on an absolute neighborhood extensor for metric spaces.

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1. Introduction

Recently, K. Morita [4] has introduced the following idea. Let $X$ be a topological space and $\{A_\alpha\}$ a closed covering of $X$. Then $X$ is said to have the weak topology with respect to $\{A_\alpha\}$, if the union of any subcollection $\{A_\beta\}$ of $\{A_\alpha\}$ is closed in $X$ and any subset of $\bigcup_\beta A_\beta$ whose intersection with each $A_\beta$ is closed relative to the subspace topology of $A_\beta$ is necessarily closed in the subspace $\bigcup_\beta A_\beta$.

E. Michael [3] has introduced the following notion. A topological space $X$ is called an absolute extensor (resp. absolute neighborhood extensor) for metric spaces if, whenever $Y$ is a metric space and $B$ is a closed subset of $Y$, then any continuous mapping from $B$ into $X$ can be extended to a continuous mapping from $Y$ (resp. some neighborhood of $B$ in $Y$) into $X$. A topological space $X$ is called an absolute retract (resp. absolute neighborhood retract) for metric spaces if, whenever $X$ is a closed subset of a metric space $Y$, there exists a continuous mapping from $Y$ (resp. some neighborhood of $Y$ in $X$) onto $X$ which keeps $X$ pointwise fixed. We shall use the following abbreviations as Michael [3]:

AE = absolute extensor.
ANE = absolute neighborhood extensor.
AR = absolute retract.
ANR = absolute neighborhood retract.

The purpose of this paper is to establish the following theorem.

Theorem. Let $X$ be a topological space having the weak topology with respect to a closed covering $\{A_\alpha\}$. We assume that, for each finite subcollection $\{A_{\alpha_1}, A_{\alpha_2}, \ldots, A_{\alpha_n}\}$ of $\{A_\alpha\}$ with non-void intersection, $\bigcap_{i=1}^{n} A_{\alpha_i}$