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Analogue of a theorem of F. and M. Riesz for minimal surfaces.

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In a fundamental paper published some years ago, Beckenbach and Radó showed how certain function-theoretic methods and results could be extended to obtain analogous results for functions of class *PL*; they also showed how the new results could be utilized to obtain additional theorems relating to minimal surfaces [1].

In that same spirit, we use a function-theoretic technique to obtain an analogue for functions of class *PL*, of the well-known theorem of F. and M. Riesz [2; p. 46]; then we obtain the corresponding result for minimal surfaces. It should be noted that our first theorem could also be obtained from the deep results due to Littlewood [4] and Deny and Lelong [3].

Let $p(z)\equiv p(x,y)$ be a real-valued function defined for z in the unit disc $\mathfrak{D}: |z|<1$. Then $p(z)$ is said to be of class *PL* if and only if the following conditions hold [1; p. 651]: (i) $p(z)$ is continuous, (ii) $p(z)\geqq 0$, and (iii) $\log p(z)$ is subharmonic in that part of $\mathfrak{D}$ where $p(z)>0$.

For functions of class *PL* we have the following result, which generalizes a theorem due to Beckenbach and Radó [1; p. 652].

THEOREM 1. *Let $p(z)$ be of class PL and bounded in $\mathfrak{D}$, and let $E\equiv[\theta \mid \lim_{r\to 1} p(re^{i\theta})=0]$, $z=re^{i\theta}$. If the (linear) measure mE of E is positive, then $p(z)\equiv 0$.*

PROOF. First, E is measurable. Moreover, if $mE=2\pi$, then it follows from Lebesgue's theorem that

$$\lim_{r\to 1}\int_0^{2\pi} p(re^{i\theta})\,d\theta=\int_0^{2\pi}\lim_{r\to 1} p(re^{i\theta})\,d\theta=0\,. \tag{1}$$

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