

## On the fundamental conjecture of GLC IV.

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This paper belongs to the series of papers [2], [3], [4]. In [2] the author has proved the following theorem:

The end-sequence of a normal proof-figure in  $G^1LC$  is proved without cut.

The logical system  $G^1LC$  is a subsystem of  $GLC$  defined in [1], where we have enounced the "fundamental conjecture" that every provable sequence in  $GLC$  would be provable without cut. In this paper we shall generalize the above result of [2] in proving a theorem of the same form in  $GLC$ , when the meaning of "normal" is also widened than in [2] (even restricted to the case of  $G^1LC$ ). We shall prove this result in Chap. II after preparations in Chap. I. At the end of the paper, we shall also prove a lemma (as Lemma 2) which we have used in [4] without proof.

### Chapter I. The proof-figure of $GLC$

The whole paper is based on  $GLC$  as was explained in [1], chapter I. However we shall modify some notions as follows.

#### § I. Symbols

As in [1], we use the following symbols:

##### 1.1. Variables

##### 1.1.1. $t$ -variables ( $t$ means 'term')

1.1.1.1.  $t$ -variables without argument-place, which is called variables of type (0) in [1].

Free ones:  $a_0, b_0, c_0, \dots$

Bound ones:  $x_0, y_0, z_0, \dots$

(In this paper, we have not to distinguish special  $t$ -variables and special  $f$ -variables, among free  $t$ -variables and free  $f$ -variables in general.)