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On the Poisson distribution.

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Let $\cdots, x_{-1}, x_0, x_1, \cdots$ be the points on the real line such that $\cdots x_{-1} < x_0 < x_1 \cdots (x_0 \equiv 0)$. Then if $\{x_j - x_{j-1}\}$ $(j = 0, 1, 2, \cdots)$ are independent random variables with common distribution function $F(x)$, where $F(x)$ is the distribution function of a non-negative random variable with $F(-0) = 0$, $F(\infty) = 1$, we shall say that these points are distributed at random according to $F(x)$.

Now consider a system of particles $P_n (n = 0, \pm 1, \pm 2, \cdots)$ which start from the above stated random positions $x_n (n = 0, \pm 1, \pm 2, \cdots)$. When we denote by $X_n(t)$ the displacement of the n-th particle P_n up to the time t, the coordinate $Y_n(t)$ of the particle at the time t is

$$Y_n(t) = x_n + X_n(t), \qquad X_n(0) = 0, \qquad t \geqq 0.$$

In the following, let us confine ourselves to the discrete time parameter $t = 0, 1, 2, \cdots$, and we shall impose the following conditions on the movement of the particles. The random variables $X_n(t) - X_n(t-1)$ are mutually independent for each n, t, $-\infty < n < \infty$, $t \geqq 0$, and obey the same distribution function $G(x)$ for all n, t, moreover, for each $t > 0$ the classes of random variables

$$\{X_n(t), n = 0, \pm 1, \pm 2, \cdots\} \qquad \{x_n, n = 0, \pm 1, \pm 2, \cdots\}$$

are mutually independent.

By the Fourier analytical method [2], Prof. Maruyama [3] investigated the limiting distribution of the number $N_I(t)$ of particles lying in an interval $I = [a, b]$ at t under the condition that $G(x)$ is a non-lattice distribution function. In this note, we shall discuss the problem when $G(x)$ is a lattice distribution function with maximum span $d > 0$.

THEOREM. *If* $0 < m = \int_{-\infty}^{+\infty} x dF(x) < \infty$, *then we have*