Journal of the Mathematical Society of Japan Vol. 8, No. 2, April, 1956

Harmonic analysis of the axially symmetrical incompressible viscous flow.

By Prem PRAKASH

(Received Dec. 8, 1955)

Introduction : The present paper has been inspired by a similar one for the two-dimensional case*. For an axially symmetrical three-dimensional flow, the paper gives the differential equations connecting the Fourier transforms of the velocity components, the stream function and the vorticity. Equivalent integro-differential equation of the Navier-Stokes equation of motion has been given and thus the Fourier transform Z of the vorticity being known, those of the velocity components and the stream function are given; also the spectral function of the kinetic energy of the flow is determined.

Flow being axially symmetrical it will be the same in all the planes passing through the axis. We therefore consider the flow in half of one of these planes bounded by a domain D in this plane, velocity vanishing on the boundary B of D. We take x along the axis and ϖ perpendicular to it as the coordinates of any point in this half-plane. If the fluid extends up to infinity, the velocity is zero there. If ψ be the stream function the two components u, v of the velocity are given by

$$u(x, \varpi, t)=-\frac{1}{\varpi}\frac{\partial\psi}{\partial\varpi}, \tag{1}$$

$$v(x, \varpi, t)=\frac{1}{\varpi}\frac{\partial\psi}{\partial x}. \tag{2}$$

According to boundary conditions we have

* J. Kampé de Fériet, Harmonic analysis of the two-dimensional flow of an incompressible viscous fluid, Quart. Appl. Math., (1) VI, 1–13 (1948).