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Geodesic correspondence of Riemann spaces.

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It is well known that the central projection of a sphere on a plane induces a geodesic correspondence between the sphere and the plane, while the stereographic projection induces a conformal correspondence between them. In the present paper we define characteristic roots of a conformal correspondence between Riemann spaces and show that when all the characteristic roots are equal and some additional conditions are satisfied, two spaces have some special properties concerning geodesic and conformal correspondences, which can be regarded as a generalization of the case of sphere and plane. Throughout the paper our treatment is local and we follow the convention that the repeated indices imply summation and assume that the indices i, j, k, h run from 1 to n unless otherwise stated.

§ 1. Characteristic roots of a geodesic correspondence.

1. We consider two n-dimensional affinely connected spaces S and $\bar{S}$, which are both without torsion and are locally homeomorphic in such a way that the geodesics in the two spaces correspond to each other. Let the connections of S and $\bar{S}$ be respectively defined by

$$dA=\omega^i e_i\,,\qquad de_i=\omega_i^j e_j\,,$$

$$dA=\bar{\omega}^i e_i\,,\qquad de_i=\bar{\omega}_i^j e_j\,.$$

Since there are no torsions we have

$$d\omega^i=\omega^j\wedge\omega_j^i\,,\qquad d\bar{\omega}^i=\bar{\omega}^j\wedge\bar{\omega}_j^i\,. \tag{1}$$

On account of the local homeomorphism we can take frames in the tangent spaces in such a way that we have