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Additive prime number theory in an algebraic number field.

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Thanks to the remarkable work of Vinogradov [7], we know that every sufficiently large odd integer can be expressed as a sum of three primes. Less attention has been paid to the problem of representing numbers in an algebraic number field as a sum of primes. Rademacher [4] carried over the Hardy-Littlewood formula in the rational case to a real quadratic number field on a certain hypothesis concerning the distribution of the zeros of Hecke's $\zeta(s, \lambda)$ funcfions.

Let K be an algebraic number field of degree n with r_1 real conjugates $K^{(l)}$ $(l=1, 2, \cdots, r_1)$ and r_2 pairs of conjugate complex conjugates $K^{(m)}, K^{(m+r_2)}$ $(m=r_1+1, r_1+2, \cdots, r_1+r_2)$ so that $r_1+2r_2=n$. Let a, b be positive and μ, ν be in K. For convenience, we use the symbol

$$a\|\mu\| \leqq b\|\nu\|$$

in the sense that

$$a|\mu^{(i)}| \leqq b|\nu^{(i)}| \qquad (i=1, 2, \cdots, n).$$

For example, $\|\mu\| \leqq b$ means $|\mu^{(i)}| \leqq b$. Let $\mathfrak{a}$ be any principal ideal in K. By the theory of units, there exist a positive constant c_0 depending only on K and at least one ν in K such that

$$\text{(1)} \qquad \mathfrak{a}=(\nu) \quad \text{and} \quad \|\nu\| \leqq c_0 \sqrt[n]{N(\nu)}.$$

In what follows we fix this constant c_0. We use a letter c to denote a positive constant depending only on K, not necessarily the same each time it occurs. The symbol

$$Y=O(X)$$