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On the fundamental conjecture of *GLC* II.

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This paper is a continuation of our former papers [1], [2]. In [1] we have extended Gentzen's logic calculus *LK* (Gentzen [3]) to *GLC* (generalized logic calculus) and shown that the "fundamental conjecture" of *GLC* would mean the consistency of the analysis. Here and in the following, we shall mean by the "fundamental conjecture" (abbrev. *FC*) of a logical system the theorem of the type: Every provable sequence in the logical system in question is provable without cut in the same logical system. Now G^1LC is a subsystem of *GLC* obtained from *LK* by introducing *f*-variables with n argument-places $\alpha[*_1,\ldots,*_n]$ $(n\geq 1)$ (for terminology see [2]). It was also shown in [1] that *FC* of G^1LC would imply the consistency of the theory of real numbers. In [2] we have proved *FC* on a certain subsystem of G^1LC (containing of course *LK* of Gentzen), from which the consistency of the theory of natural numbers follows.

The purpose of the present paper is to prove *FC* on two more subsystems of G^1LC, which will be called *PL* and *QL*. *PL* is obtained from *LK* by introducing *f*-variables without argument-place $\alpha, \beta, \ldots$ *) *QL* is a subsystem of G^1LC containing only proof-figures having no inference $\forall$ on variables of type (0).

We give the proof only for the case of *QL*, but it is easy to see that our proof holds also for *PL;* one has only to put the number n of argument-places of *f*-variables to 0 everywhere in the proof, and to notice that the essential point of the proof depends on the following circumstance. Suppose there appears an inference $\forall$ on *f*-variable in a proof-figure. We shall denote with $\mathfrak{J}$ this inference.

*) *f*-variables without argument-place $\alpha, \beta, \ldots$ may be considered as $\alpha'[1], \beta'[1], \ldots$, where $\alpha', \beta', \ldots$ are *f*-variables with one argument-place and 1 is a fixed special variable. Thus *PL* is a subsystem of G^1LC.