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On the family of connected subsets and the topology of spaces.

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Let A be a set of points. If we introduce a topology into A, then the family $\{C\}$ of all connected subsets of A will be determined. But it is easily known by a simple example that the topology introduced into A can not be determined in general by $\{C\}$. But under some conditions it happens that the topology of the space can be determined by $\{C\}$, as will be shown in the following.

In this paper we shall use the notions and terminologies in G. T. Whyburn's "Analytic topology" [1]. We shall assume that all spaces to be considered are separable metric spaces. We begin with the definition of a biconnected transformation which plays an important role in this paper.

DEFINITION. *Let A and B be spaces. A transformation f of A onto B will be called a biconnected transformation if the following conditions are satisfied:*

(1) *f is one-to-one.*

(2) *Connected subsets of A are transformed to connected subsets of B under f, and conversely.*

Then we consider the properties of A and B under which the biconnected transformation f becomes a continuous or topological transformation.

THEOREM 1. *Let f be a biconnected transformation of A onto B, where A is a space and B is a semi-locally-connected space*[1]. *Then f is a continuous transformation.*

PROOF. Suppose, on the contrary, that f is not continuous at a point p of A. Let $\{p_n\}$ be a sequence of points of A converging to p such that $\{f(p_n)\}$ does not converge to $f(p)$. Since B is semi-locally-connected, there exists a neighborhood U of $f(p)$ such that $B-U$ contains an infinite number of points of $\{f(p_n)\}$ and consists of