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On n-dimensional homogeneous spaces of Lie groups of dimension greater than $n(n-1)/2$.

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0. Introduction

The purpose of this note is to determine the Lie groups of dimension greater than $n(n-1)/2$ which can be treated as groups of isometries on an n-dimensional Riemannian space and study the differential-geometrical and topological structure of the space.

In this regard, K. Yano [5] has recently proved the following interesting theorem.

THEOREM A. *A necessary and sufficient condition that an n-dimensional Riemannian space for $n>4$, $n\neq 8$ admit a group of motions of order $n(n-1)/2+1$ is that the space be the product space of a straight line and an $(n-1)$-dimensional Riemannian space of constant curvature or that the space be of negative constant curvature.*

In this theorem the cases $n=4$ and $n=8$ are exceptional. For $n=4$, S. Ishihara [1] has solved the problem completely by determining all 4-dimensional homogeneous Riemannian spaces, but it was open for $n=8$.

On the other hand, to prove Theorem A, K. Yano used essentially the following theorem due to D. Montgomery and H. Samelson [3].

THEOREM B. *The rotation group $R(n)$ in an n-dimensional vector space, for $n\neq 4$, $n\neq 8$, contains no proper closed subgroup whose dimension is greater than $(n-1)(n-2)/2$. If H is a subgroup whose dimension is equal to $(n-1)(n-2)/2$, then H is the subgroup which leaves fixed one and only one direction.*

As to the case $n=8$, it has already been known that $R(8)$ contains the universal covering group of $R(7)$[1]. This implies that the

1) Prof. S. Murakami has kindly informed me this fact and others concerned. I should like to express my hearty thanks to him.