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Borel's direction of a meromorphic function in a unit circle.

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1. Analogue of Biernacki-Rauch's theorem.

Let $f(z)$ be a meromorphic function of finite order $\rho>0$ for $|z|<\infty$, then Valiron[1)] proved that there exists a Borel's direction J: $\arg z=\theta_0$, which satisfies the following condition. Let ω: $|\arg z-\theta_0|<\delta$ be any small angular domain, which contains J and $z_\nu(a, \omega)$ be zero points of $f(z)-a$ in ω, multiple zeros being counted only once, then for any $\varepsilon>0$,

$$\sum_\nu \frac{1}{|z_\nu(a, \omega)|^{\rho-\varepsilon}}=\infty$$

with two possible exceptions for a.

If $f(z)$ is of divergence type, then

$$\sum_\nu \frac{1}{|z_\nu(a, \omega)|^{\rho}}=\infty$$

with two possible exceptions for a.

This Valiron's theorem is generalized by Biernacki and Rauch as follows.

Let $g(z)$ be a meromorphic function of order $<\rho$ for $|z|<\infty$, and $z_\nu(f=g, \omega)$ be zero points of $f(z)-g(z)$ in ω, multiple zeros being counted only once, then for any $\varepsilon>0$,

$$\sum_\nu \frac{1}{|z_\nu(f=g, \omega)|^{\rho-\varepsilon}}=\infty$$

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1) G. Valiron: Recherches sur le théorème de M. Borel dans la théorie des fonctions méromorphes. Acta Math. 52 (1928). M. Tsuji: On Borel's directions of moromorphic functions of finite order. Tohoku Math. Journ. 2 (1950).