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A remark on my former paper "Theory of Fuchsian groups".

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Let G be a Fuchsian group of linear transformations, which make $|z|<1$ invariant and D_0 be its fundamental domain. We denote its part, contained in $|z|<\rho<1$ by $D_0(\rho)$. We define the non-euclidean line- and surface-element by

$$ds=\frac{2|dz|}{1-|z|^2}, \quad d\sigma=\frac{4\,dxdy}{(1-|z|^2)^2}, \quad z=x+iy. \tag{1}$$

Let $a\in D_0$ and we denote its equivalents by a_ν ($\nu=0,1,2,\cdots, a_0=a$) and $n(r,a)$ be the number of a_ν, contained in $|z|<r<1$ and put

$$N(r,a)=\int_{\frac{1}{2}}^{r}\frac{n(r,a)dr}{r}. \tag{2}$$

Let $a\in D_0, b\in D_0$ $(a\neq b)$, then there exists a potential function $u(z;a,b)$, which is invariant by G and is harmonic in $|z|<1$, except at a_ν, b_ν, where

$$u(z;a,b)-\log\frac{1}{|z-a_\nu|}, \quad u(z;a,b)+\log\frac{1}{|z-b_\nu|}$$

are harmonic.

Let $u^+=u$, if $u\geqq 0$, $u^+=0$, if $u\leqq 0$, and put for a fixed b

$$m(r,a)=\frac{1}{2\pi}\int_0^{2\pi}u^+(re^{i\theta};a,b)d\theta, \tag{3}$$

$$T(r,a)=m(r,a)+N(r,a). \tag{4}$$