Journal of the Mathematical Society of Japan Vol. 6, Nos. 3～4, December, 1954

On a conjecture of Kaplansky on quadratic forms

By Tosirô TSUZUKU

(Received April 20, 1654)

In his recent paper[1] Kaplansky took up some problems on quadratic forms over a not formally real field of characteristic different from two. Among others he made the following conjecture: Let F be a field of characteristic different from two which is not formally real, and let the multiplicative group of non-zero elements of F modulo squares be pricisely of order n. Then every quadratic form in $n+1$ variables over F represents zero (non-trivally). He affirmed this conjecture in the following two special caces: (1) $n\leqq 8$, (2) -1 is a sum of four or less squares in F. In the present paper we shall show on modifying and refining Kaplansky's methods that his conjecture is true; in fact we shall prove a more finer statement.

The writer wishes to express his gratitude to Prof. T. Nakayama and Mr. T. Ono for their valuable suggestions.

Let F be a field of characteristic different from two which is not formally real (that is, -1 is a sum of squares in F). We shall fix this field throughout this paper. After Kaplansky, we define three invariants of F as follows;

(a) A is the order of the multiplicative group of non-zero elements of F moduls squares. A may be infinite; if it is finite it is evidently a power of 2.

(b) B is the smallest integer n such that -1 is a sum of n squares in F.

(c) C is the smallest integer n such that every quadratic form in $n+1$ variables over F is a null form (i.e. a form which represents zero non-trivially).

On the value of B, we have the following

1) I. Kaplansky, "Quadratic forms" J. Math. Soc. Japan, vol. 5 (1953) pp. 200-207. We refer of this paper as K. Q.