Journal of the Mathematical Society of Japan Vol. 6, Nos. 3~4, December, 1954

The boundary distortion on conformal mapping.

By Masatsugu TSUJI

(Received Feb. 6, 1954)

CONTENTS

1. Main Theorems.

1. Let D de a domain on the $w=\xi+i\eta$-plane, which is bounded by a Jordan curve C, which passes through $w=0$ and touches the real axis at $w=0$ and its inner normal at $w=0$ coincides with the positive η-axis. We map D conformally on the upper half $\Im z>0$ of the $z=x+iy$-plane by $w=w(z)$, $w(0)=0$. There are many researches concerning the existence of $\lim\limits_{z\to 0}\dfrac{w(z)}{z}$. Among others, we state the following theorems.

THEOREM 1. (*Carathéodory*)[1]. *If there are two circles K_1, K_2, which touch the real axis at $w=0$, such that K_1 lies in D and K_2 lies outside of D, then*

$$\lim_{z\to 0}\frac{w(z)}{z}=\lim_{z\to 0}w'(z)=\gamma\,,\qquad 0<\gamma<\infty\,,$$

exists uniformly, when $z\to 0$ in any Stolz domain, whose vertex is at $z=0$.

THEOREM 2. (*Besonoff-Lavrentieff*)[2]. *If in a neighbourhood of $w=0$,* (i) *C lies between two curves:*

$$H:\ \eta=|\xi|^{1+\alpha}\quad \textit{and}\quad \overline{H}:\ \eta=-|\xi|^{1+\alpha}\quad (0<\alpha<1)$$

1) C. Carathéodory: Über die Winkelderivierte von beschränkten analytischen Funktionen. Sitzber. der Berl. Akad. 1929.

2) P. Besonoff et M. Lavrentieff: Sur l'existence de la derivée limite. Bull. Soc. Math. 58 (1930).