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On some matrix operators.

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0. Introduction.

Let K be an arbitrary field of any characteristic $\chi(K)\,(=0$ or $p)$. We denote by $\mathfrak{gl}(K, n)$ the set of all matrices of degree n over K and by $GL(K, n)$ the set of all non-singular matrices in $\mathfrak{gl}(K, n)$. I_n and O_n mean the unit matrix and zero matrix of degree n respectively.

Besides ordinary operations on matrices, we consider the following three operators. For $A=(a_{ij})\in\mathfrak{gl}(K, n)$ and $B\in\mathfrak{gl}(K, m)$ we consider the direct sum:

$$A\dot{+}B=\begin{pmatrix} A & O \\ O & B \end{pmatrix}\in\mathfrak{gl}(K, n+m),$$

the Kronecker product:

$$A\otimes B=\begin{pmatrix} a_{11}B, a_{12}B, \cdots, a_{1n}B \\ \\ a_{n1}B, a_{n2}B, \cdots, a_{nn}B \end{pmatrix}\in\mathfrak{gl}(K, nm),$$

and the Kronecker sum: $A\oplus B=A\otimes I_m+I_n\otimes B\in\mathfrak{gl}(K, nm)$.
These operations $\dot{+}, \otimes, \oplus$ are non-commutative but associative.
Now we define two set-theoretical sums:

$$\mathfrak{R}=\mathfrak{R}(K)=\bigcup_{n=1}^{\infty}\mathfrak{gl}(K, n), \qquad \mathfrak{S}=\mathfrak{S}(K)=\bigcup_{n=1}^{\infty}GL(K, n).$$

For an element A in $\mathfrak{R}$, we denote by $d(A)$ its degree.

Now let L be a Lie algebra over K and $\mathfrak{R}_0\ni\rho_1, \rho_2, \cdots$ the set of representations of L. Between the elements of $\mathfrak{R}_0$, the operations such as $\rho_1\dot{+}\rho_2$, $\rho_1\oplus\rho_2$ are defined in the well-known way. We can also speak of the degree $d(\rho)$ of ρ, and of the transform $T\rho T^{-1}$ of ρ by an element T in $GL(K, d(\rho))$.

Harish-Chandra [1] has considered a mapping ζ of $\mathfrak{R}_0$ into $\mathfrak{R}$, satisfying the following conditions: