Journal of the Mathematical Society of Japan Vol. 6, No. 1, March, 1954

On Royden's theorem on a covering surface of a closed Riemann surface.

By Masatsugu TSUJI

(Received Nov. 4, 1953)

Let F be a closed Riemann surface of genus $p\geqq 2$, spread over the z-plane and Φ be its unramified covering surface. Let C_i $(i=1,2,\cdots,p)$ be p disjoint ring cuts of F, such that, if we cut F along $\{C_i\}$, then F becomes a surface of planar character and let C_i' be the conjugate ring cut of C_i, such that C_i' meets C_i at a point and is disjoint to C_j, C_j' $(j=1,2,\cdots,p,\ j\neq i)$. We assume that C_i, C_i' are rectilinear polygons and meet at a positive angle. We denote the both shores of C_i, C_i' by $C_i^+, C_i^-, C_i'^+, C_i'^-$ respectively.

We denote a surface, which is obtained from F by cutting along a certain number of C_i, C_i' by F' in general, then

$$\Phi=\sum_{k=0}^{\infty} F_k',$$

where F_k' is one F'.

Let Γ_k be the boundary of F_k', which consists of a certain number of $C_i^+, C_i^-, C_i'^+, C_i'^-$, which we denote by $\{\sigma_k^{(i)}\}_{i=1,2,\cdots,\alpha_k}$ so that $\Gamma_k=\sum_i \sigma_k^{(i)}$. Along $\sigma_k^{(i)}$, there connects another F_s' to F_k'.

Then Royden[1)] proved the following theorem.

THEOREM. *The necessary and sufficient condition that Φ is of positive boundary is that there exist a contant $m_k^{(i)}$ corresponding to $\sigma_k^{(i)}$, such that if $\sigma_k^{(i)}$ belong to the boundary of another F_s' and $\sigma_k^{(i)}=\sigma_s^{(j)}$, then $m_s^{(j)}=-m_k^{(i)}$ and satisfy the following conditions:*

(i) $$\sum_i m_0^{(i)}\neq 0, \qquad \sum_i m_k^{(i)}=0 \quad (k=1,2,\cdots),$$

(ii) $$\sum_{k=0}^{\infty} M_k^2<\infty,$$

1) H. L. Royden: Harmonic functions on open Riemann surfaces. Trans. Amer. Math. Soc. 75 (1952).