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On the normal form of cohomology groups.

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In a previous note[1] I have obtained an extension of the well-known Hilbert's norm theorem concerning cyclic fields to the case of abelian extensions, but only for the case of p-adic number fields as ground fields. The present investigation started at its first stage to fill up this lack, but the problem was quite difficult to overwhelm, and the results we have obtained have an entirely different meaning.

Instead of the factor sets treated in 1), we take up generally the cochain of degree n, with finite abelian operator domain.

For the case of $n=2$, O. Schreier has obtained a normal form for the factor sets, in his famous research on group extension.[2] A part of this result is generalized by Lyndon,[3] and we shall reproduce the results by another approach. We then give a more precise normal form, which is a direct generalization of the fact, that the Galois-cohomology group of dimension 2 by a cyclic field is isomorphic to the norm residue class group.

The main principle which governs this paper lies in the concept of splitting groups, which have slightly weakend property compared with Artin's splitting group.

Several parts of our investigation were also obtained by S. Takahashi independently and appeared in a recent number of Tohoku Mathematical Journal.[4]

§1. Throughout this paper we treat the cocycles with values in an abelian group Ω and with an abelian operator domain G. We call a cocycles to be splitting or bounding, if it is a bounding cocycle. Then we have the following (weakend) analogy of Artin's splitting group, which is known as the reduction theorem.

THEOREM 1. *To each cocycle f, we find an extension Ω^* of Ω, in which f splits.*

PROOF. For the sake of brevity, we shall assume that the dimension n of f is 3. We have by the assumption