

On the semi-groups generated by Kolmogoroff's differential equations.

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§ 1. Introduction.

Let $P(t) = (p_{jk}(t))$, $0 \leq t < \infty$, be the matrix of transition probabilities of a temporally homogeneous Markoff process with denumerable states. $P(t)$ has the properties:

$$(1.1) \quad p_{jk}(t) \geq 0,$$

$$(1.2) \quad \sum_{k=1}^{\infty} p_{jk}(t) = 1,$$

$$(1.3) \quad p_{ik}(s+t) = \sum_{j=1}^{\infty} p_{ij}(s) p_{jk}(t).$$

It is also known¹⁾ that, under certain general conditions, $p_{jk}(t)$ satisfy the following Kolmogoroff's differential equations:

$$(1.4) \quad dp_{ik}(t)/dt = \sum_{j=1}^{\infty} p_{ij}(t) a_{jk},$$

$$(1.4') \quad dp_{ik}(t)/dt = \sum_{j=1}^{\infty} a_{ij} p_{jk}(t),$$

together with the initial condition

$$(1.5) \quad \lim_{t \rightarrow 0} p_{jk}(t) = p_{jk}(0) = \delta_{jk} = \begin{cases} 1, & j=k, \\ 0, & j \neq k. \end{cases}$$

Here a_{jk} are constants satisfying the conditions

1) For example Feller [6], Doob [4], Hille [7].