Journal of the Mathematical Society of Japan Vol. 5, Nos. 3～4, November, 1953.

Generalized evolute in Klein spaces.

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(Received Feb. 11, 1953)

We investigate in this paper the generalization of the enveloping theorem of an evolute of a curve on the euclidean plane to the case of figures in Klein spaces by the method of moving frame of E. Cartan [1]. The idea of this paper is the same with that of [3]. In addition we state the process of obtaining theorems in Klein spaces analogous to Euler-Savary's theorem on the euclidean plane ([2] pp. 28–29).

1. Generalized evolute

1.1 Let $\mathfrak{G}$ be a fundamental Lie group of the Klein space and $\mathfrak{H}$ a closed subgroup of $\mathfrak{G}$. We consider the figure F consisting of one-parametric set of points of the homogeneous space $\mathfrak{G}/\mathfrak{H}$ and attach to each element of F a Frenet's frame defined in [1] pp. 131–132. Let the Frenet's frame defined at the point A on F be $S_a R$, where R is a fundamental frame and S_a is an element of $\mathfrak{G}$, and let the Frenet's frame at a consecutive point of F be $S_{a+da}R$. The frames whose relative displacements are each given by S_t with respect to $S_a R$ and $S_{a+da}R$ are $S_a S_t R$ and $S_{a+da} S_t R$. The infinitesimal relative displacement between $S_a S_t R$ and $S_{a+da} S_t R$ is given by $(S_a S_t)^{-1}(S_{a+da} S_t) = S_t^{-1}(S_a^{-1} S_{a+da}) S_t$. We take S_t which depends on the parameter a, so that $S_t^{-1}(S_a^{-1} S_{a+da}) S_t$ is an infinitesimal element of a certain fixed subgroup $\mathfrak{K}$ of $\mathfrak{G}$ for all a. $\mathfrak{K}$ is not in general unique. We call the elements of the homogeneous space $\mathfrak{G}/\mathfrak{K}$ belonging to $S_a S_t R$ a central figure. To each point of F a central figure is defined and we call a set of central figures an evolute of F, which we denote by E. The infinitesimal relative displacement of the frames $S_a S_t R$ attached to E can be given by

$$(S_a S_t)^{-1}(S_{a+da} S_{t+dt}) = S_t^{-1}(S_a^{-1} S_{a+da})\, S_t \cdot S_t^{-1} S_{t+dt}\,.$$

Let the relative components of the relative displacement of $S_a S_t$, S_a, S_t be $\omega_p, \omega_p^{(1)}, \omega_p^{(0)}$ $(p=1, 2, \cdots, r)$ respectively. Then we have the relations