Journal of the Mathematical Society of Japan Vol. 5, Nos. 3~4, November, 1953.

On the exceptional set of a certain harmonic function in a unit sphere.

By Masatsugu TSUJI.

(Received May 28, 1953)

1. Main theorem.

In the former paper[1], I have proved, by generalizing Beurling's theorem[2], the following theorem.

THEOREM 1. *Let $f(z)$ be regular in $|z|<1$ and*

$$\iint_{|z|<1} |f'(z)|^2 dxdy < \infty, \quad z=x+iy.$$

Then there exists a set E on $|z|=1$, which is of logarithmic capacity zero, such that if $e^{i\theta}$ does not belong to E, then

$$\lim_{z\to e^{i\theta}} f(z)=f(e^{i\theta})\ (\neq\infty) \text{ exists and uniformly,}$$

when z tends to $e^{i\theta}$ from the inside of any Stolz domain, whose vertex is at $e^{i\theta}$ and for any rectilinear segment l, which connects $e^{i\theta}$ to a point of $|z|<1$,

$$\int_l |f'(z)||dz|<\infty.$$

From this, we have

THEOREM 2. *Let $u(z)$ be harmonic in $|z|<1$ and*

$$\iint_{|z|<1} |\operatorname{grad} u(z)|^2 dxdy < \infty.$$

Then there exists a set E on $|z|=1$, which is of logarithmic capacity zero, such that if $e^{i\theta}$ does not belong to E, then

$$\lim_{z\to e^{i\theta}} u(z)=u(e^{i\theta})\ (\neq\infty) \text{ exists and uniformly,}$$

1) M. Tsuji: Beurling's theorem on exceptional sets. Tohoku Math. Journ. 2 (1950).
2) A. Beurling: Ensembles exceptionels. Acta Math. 72 (1940).