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On mixed boundary value problems for functions analytic in a simply-connected domain

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1. Formulation of problem.

In the preceding papers[1)] we have dealt with a mixed boundary value problem in potential theory. In case the unit circle laid on the z-plane is taken as the basic domain, the previous problem has been formulated as follows: To determine a function $u(z)$ harmonic and bounded in the unit circle $|z|<1$ and satisfying the boundary conditions

$$\begin{aligned} u(e^{i\varphi}) &= U_j(\varphi) \quad \text{for} \quad a_j<\varphi<b_j, \\ \frac{\partial u(e^{i\varphi})}{\partial \nu} &= V_j(\varphi) \quad \text{for} \quad b_j<\varphi<a_{j+1} \end{aligned} \qquad (j=1,\cdots,m),$$

a_{m+1} being supposed to be coincident with $a_1+2\pi$ and $\partial/\partial\nu\equiv\partial/\partial\nu_\varphi$ denoting the differentiation along the inward normal at $e^{i\varphi}$. The prescribed boundary functions $U_j(\varphi)$ and $V_j(\varphi)$ are supposed, for instance, continuous and bounded over their respective intervals of definition.

It has been shown that the solution of the problem is surely existent and uniquely determined and further that it can be represented by the integral formula

$$u(z)=\frac{1}{2\pi}\sum_{j=1}^{m}\left\{\int_{a_j}^{b_j} U_j(\varphi)\frac{\partial}{\partial\nu}\lg|\Phi(e^{i\varphi},z)|\,d\varphi - \int_{b_j}^{a_{j+1}} V_j(\varphi)\lg|\Phi(e^{i\varphi},z)|\,d\varphi\right\}.$$

Here, $\Phi(\zeta,z)$ denotes the function mapping $|\zeta|<1$ schlicht and conformally onto the exterior of the unit circle cut along m radial slits starting orthogonally at points on the unit circumference in such a way