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An imbedding theorem on finite covering surfaces of the Riemann sphere.

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Let Δ be a finite covering surface of the Riemann sphere Σ, i.e. a covering surface consisting of a finite number of closed triangles. Then, it was proved in [1] that *Δ can be imbedded in a closed covering surface of Σ*. According to the construction described therein, however, the genus of the resulting closed surface is in general higher than that of Δ. In § 1 of the present paper, we shall prove:

A finite covering surface Δ of Σ can be imbedded in a closed covering surface of the same genus.

In § 2, an analogous theorem concerning analytic differentials is stated and proved. Finally, in § 3, an application is made to the theory of open Riemann surfaces.

1. It is a trivial fact that, as an abstract Riemann surface, Δ can be imbedded in a closed Riemann surface of the same genus. In this connection, our theorem may be formulated in the following form:

THEOREM 1. *Let D be a subregion of a Riemann surface F, such that the closure $\bar{D}$ of D is compact and that the boundary of D consists of a finite number of Jordan curves. Let $f(p)$ be a function defined and analytic on $\bar{D}$ (poles being admitted). Then, $\bar{D}$ can be imbedded in a closed Riemann surface D^* of the same genus as D, in such a manner that $f(p)$ can be continued to a function defined and analytic on D^*.*

PROOF. We assume that $f(p) \not\equiv$ const., since, otherwise, any closed prolongation of D of the same genus has the required property. Further, we may assume that $f(p)$ is defined and analytic throughout F. If the values of $f(p)$ are represented by points on the Riemann sphere Σ, F is mapped by $f(p)$ onto a covering surface Φ of Σ. The image