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Quadratic forms.

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1. Introduction. The theory of quadratic forms over a p-adic number field is a vital building block of the Minkowski-Hasse theory of quadratic forms over an algebraic number field, and has been expounded from several different points of view. A recent account of the purely number-theoretic attack on the problem appears in [2]. A field with a valuation, subjected to appropriate axioms, is the framework in [1]. A development from the point of view of the theory of algebras is given in [3].

In this last reference it appears that all the major portions of the theory can be deduced just from the fact that a quadratic form in five variables must represent zero. The main point of the present paper is that this in turn can be deduced from the following assumptions on a field F: that F is not formally real, and that its multiplicative group mod squares has exactly order four. It is plausible to make the following more general conjecture: if there are n classes mod squares, then every quadratic form in $n+1$ variables represents zero. We are able to prove this for the next case ($n=8$), and in other cases as well, for instance characteristic p. Beyond these partial results, it seems to be worth while to give a systematic formulation of the problems involved.

Notation: we shall use the symbol $(a_1, \cdots, a_n)$ for the quadratic form $\sum a_i x_i^2$. Equivalence of quadratic forms (or congruence of the corresponding matrices) will be indicated by the notation

$$(a_1, \cdots, a_n) \sim (b_1, \cdots, b_n).$$

2. Three invariants. Throughout the paper F will denote a field of characteristic different from two, and it will be assumed that F is not formally real (that is, -1 is a sum of squares). The formally real case probably has a parallel theory, which may be worth separate