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On the three-dimensional cohomology group of Lie algebras.

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Eilenberg and MacLane [1] have built up, by means of the cohomology theory, an analogue in the theory of groups to the theory of the Brauer group of normal simple algebras with a fixed splitting field over a given field: The theory of similarity classes of Q-kernels with a fixed abelian group G as center. They arrived at a remarkable result that the group of similarity classes of Q-kernels is isomorphic to the three-dimensional cohomology group $H^3(Q, G)$ of Q over the abelian coefficient group G, and gave an answer to the problem of Baer [2] on group extensions in terms of the two-dimensional cohomology theory. On the other hand, Chevalley and Eilenberg [3] have shown that the two-dimensional cohomology group $H^2(L, Z, P)$ of a Lie algebra L with respect to an abelian Lie algebra Z and a representation P of L over Z is isomorphic to the group of equivalent classes of extensions of L by (Z, P).

In the present paper, we shall try to develop further the theory of Chevalley and Eilenberg to obtain in the theory of Lie algebras an analogous result to that of Eilenberg and MacLane in the theory of groups. We shall introduce in §1 the concept of L-kernels as an analogue of Q-kernels and define the similarity group of L-kernels for Lie algebras with a fixed center. We shall show in §2 that this similarity group is isomorphic with a subgroup of the three-dimensional cohomology group $H^3(L, Z, P)$ whose meaning will be given later on, but we have not succeeded to decide whether these two groups are isomorphic with each other. In §3 we shall deal with an analogue of the problem of Baer on group extensions for Lie algebras.

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