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On the coefficients of multivalent functions.

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1. Introduction.

It has recently been conjectured by A. W. Goodman [1] that if

$$f(z)=b_1 z+b_2 z^2+\cdots+b_n z^n+\cdots \tag{1.1}$$

is regular and p-valent for $|z|<1$, then for $n>p$

$$|b_n|\leqq\sum_{k=1}^{p}\frac{2k(n+p)!}{(p+k)!(p-k)!(n-p-1)!(n^2-k^2)}|b_k|. \tag{1.2}$$

When $p=1$, this becomes the Bieberbach conjecture

$$|b_n|\leqq n|b_1|, \qquad n=2,3,\cdots \tag{1.3}$$

which has been proved for some special cases and has a long history [2]. When $p=2$ and $n=3$ (1.2) becomes

$$|b_3|\leqq 5|b_1|+4|b_2| \tag{1.4}$$

an inequality which has been proved valid, if $f(z)$ is regular 2-valent in $|z|<1$, starlike with respect to the origin, and in addition, if all b_i's are real [3].

Quite recently, by A. W. Goodman and M. S. Robertson [4], the inequality (1.2) has been proved to be valid for the class of functions called typically-real of order p, i. e. for functions with real coefficients such that $\Im f(z)$ changes its sign $2p$ times on $|z|=r$ for some range $0<\rho<r<1$.

In attempting to generalize the above results to the case where the coefficients b_n are complex, the present author was unable to obtain (1.2) for a certain class of functions to be defined in § 2, but was able to prove